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ABSTRACT

Benefit-cost analysis consists of establishing ratios of benefits to costs for a set of project variants. The decision rule is to select that project variant where the ratio is a maximum. This paper argues that specification and estimation errors can contribute to findings for large-scale systems of benefit-cost ratios approximating zero. The feasibility of descriptive, structural, and experimental approaches to benefit-cost analysis is discussed. A benefit-cost model is presented for the small-scale educational project. This model is generalized to a large-scale system, and it is demonstrated that scaleup factors such as communication and control in the organization, often overlooked, provide specification errors that contribute to the finding of negligible benefit-cost ratios. (Author)

Educational Benefit-Cost Analysis and the Problem of Scale*

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Benefit-cost analysis consists in establishing ratios of benefits to costs for a set of project variants. The decision rule is to select that project variant where the ratio is a maximum. The Coleman report on Equality of Educational Opportunity surprised many when it in essence reported the ratios for public educational projects to be of negligible magnitude. We argue that specification errors, as well as often noted estimation errors, can contribute to findings for large-scale systems of benefit-cost ratios approximating zero.

First we discuss three approaches to benefit-cost analysis, the descriptive, the structural, and the experimental approaches, and comment on the feasibility of each.(1)** A benefit-cost model is then developed for the small-scale educational project. When this model is generalized to a large-scale system, such as that of public education in an urban school district, we we invoke Kulikowski's Second Theorem to prove that scaleup factors such as communication and control in the organization, if overlooked in the benefit-cost analysis, provide specification errors which can contribute to the finding of negligible benefit-cost ratios.

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**Footnotes follow the conclusion of this paper.

The three approaches to establishing the relationship between benefits and costs that we will now discuss are descriptive cost-benefit, structural cost-benefit, and experimental cost-benefit analysis.

For a descriptive cost-benefit analysis, one has available a body of historical data, say on capital costs and estimated benefits. The costs are correlated with the benefits, usually by a multi-variate regression technique. It is important to notice that there is no concern for an educational production function in this approach. We will return to the notion of the production function in a moment. Suffice it to say here that rather than isolating factors of production in the economic sense, one merely estimates (possibly confounded) variables.[(2)] The accountant would say that the budgeting is line-item rather than functional in this approach.

A good example of a descriptive cost-benefit analysis is the well-known Equality of Educational Opportunity study. The combination of step-wise regression and non-random selection of units of analysis resulted in rather equivocal findings in that study, if one asks the relationship between variations of within school costs and variations of benefits or academic achievement.

The structural cost-benefit analysis proceeds logically (deductively) to specify a production function. On the grounds, for instance, of expert opinion, a project design is developed. This is the basis of the identification of a production function from which cost factors can be derived. The production function relates a measure of output from which benefits are estimated, to a set of factors of production. The factors of production in an urban educational project would typically include employee man-hours, the physical plant, and a curriculum and set of media. The factor costs (or descriptive data of the first approach) can be derived from the production function but not vice versa.

Finally, the experimental cost-benefit approach requires not only the identification of factors of production and functional budgeting, as in the structural approach, but also the systematic variation of these factors, once identified, to ascertain their contribution to the benefits of the project or system through time.(3) This is the fullest manifestation of what Professor Hartley aptly calls the "Output Emphasis."(4) Only with an appropriate noise reducing design and manipulation of factors can the variation of output due to each factor, be unequivocally attributed. While it is not generally feasible to utilize controlled experiments in large-scale projects,(5) this approach is widespread at the subproject or "product" level.(6)

Of the three approaches, the descriptive approach is the cheapest technique. Given the typical paucity of research resources, no other approach is feasible. ~~Hence~~ the descriptive approach is suitable, at least by default, for large-scale and national surveys. The structural approach, while potentially as expensive in undertaking as the experimental approach, requires minimal political and administrative sanction, hence is suitable for the comprehensive study of small or local projects. The experimental approach, requiring as it does elaborate sanction and support, is feasible only on the small scale.

Let us now turn to a characterization of structure in an organization such as a large urban school district. In terms of this model of structure, we can examine the effects of scale on the outcome of different approaches to benefit-cost analysis.

Let us have a set S of elements $\{x_0, x_1 \ldots x_n\}$ together with a set L of ordered pairs of distinct elements of S. Each ordered pair $(x_i, x_j) \in L$ is called a directed line y_{ij}. Suppose further that $x_i = x_j$ for the pair of lines y_{ik} and y_{jk} . Let the algebraic structure $\langle S;L \rangle$ be connected: there is no element of S which is not an element of an ordered pair of L. Finally, there exists a unique initiator element $x_0 \in S$ such that y_{i0} is not in L. Such a structure $\langle S;L \rangle$ is a tree, T.(7)

A walk in a tree is a finite sequence of elements and directed lines $\langle \ldots x_i, y_{ij}, x_j, y_{jk}, x_k \ldots \rangle$ which has an inital element x_a and a terminal element x_m. The distance d_{am} of the walk or the distance from x_a to x_m is the number of directed lines in the sequence initiated with x_a and terminated with x_m.

We can model the control or communication channels of an organization with such a tree. In particular, the initial element of the longest possible walk, or the initiator, is the manager of the organization; a directed line is a channel of communication or control; those elements which are never an initial element of any walk are subordinates; and, any elements which are initial elements (other than x_0) are intermediates. We will suppose that every subordinate has a queue which he services.

To be very specific, the subordinate will be a teacher in a large urban school system and the queue will be his pupils. It is also possible that intermediates, and even the manager, will be classroom teachers, hence service queues. The comparative value of the servicing of a given queue will be the benefit-cost ratio for that queue. We can suppose that benefits are constant across queues hence the objective function is the cost function which is to be minimized.

More formally, there is a subset $Q \subset S$, where $x \in Q$ just if x services a queue. There is a function f which maps the domain Q to a codomain $\{e\}$. Elements of $\{e\}$ are costs; the mapping is the evaluation of the servicing of a queue in terms of cost functions. Moreover, there is another function

F which maps T into a codomain $\{E\}$. The elements of $\{E\}$ are also costs; the mapping is the evaluation of the organization in terms of a cost function.

Let

$$E = \sum_{i=1}^{n} (1-\lambda_i)^{-1} e_i$$

be the aggregation model for an organization of n queues. λ_i is an index of loss in performance for the i-th queue, a loss associated with communication and control among the elements of an organization. λ_i is presumed to be a function g of the distance between the manager, who is the source of a message or command and the recipient. Thus we express the loss associated with a message from the o-th element to the j-th element as $\lambda_j = g(d\ \ )$. λ is constrained to the closed unit interval and is isotonic to the magnitude of loss.[8]

In the case of a pilot project, problems of communication and control are negligible, if we suppose that the relevant organization is a relatively small R & D center or a laboratory school where the organization is relatively "shallow." By examination of limit properties, we know that

$$\lim \sum_{i=1}^{n} (1-\lambda_i)^{-1} e_i = \sum_{i=1}^{n} e_i \lim (1-\lambda_i)^{-1} = \sum_{i=1}^{n} e_i (1 - \lim \lambda_i)^{-1}.$$

Thus $$\lim_{\lambda_i \to 0} E = \sum_{i=1}^{n} e_i \qquad (i = 1, 2 \cdots n)$$

If we additionally make D. Krathwohl's assumption of "typicality of situations,"[9] we can take the objective function of any queue as an n-th of the organizational objective function, since from

$$\frac{1}{n} \sum_{i=1}^{n} e_i = e_i$$

and the limiting case of $\lambda \to 0$, it follows that E = ne. This is the classical model of the organization, the shortcomings of which my colleague Lundin and I have criticized elsewhere.(10) Under these admittedly idealized conditions, it has just been shown that the evaluation of a single installation of a pilot program is tantamount to the assessment of the entire organization.

Let us now consider what happens to the assessments e and E in the presence of scale effects. When scaleup is undertaken, for instance, in the school district wide dissemination and installation of an educational program which presumably was successful as a pilot program in a laboratory school, problems of communication and control are no longer negligible.

Now the organization will acquire some "height," as it is plausible to assume that height and size of an organization are isotonically related.(11) It is likewise plausible to assume that height and an index of loss λ are isotonically related.(12) Then the loss function g can be further specified by an exponential function such as

$$\lambda_j = k^{d_{ij}}$$

where k is the loss in performance of a queue for messages and commands for unit distance. The organizational objective function is

$$E = \sum_{i=1}^{n} (1 - k^{d_i})^{-1} e_i$$

where d_i is the length of the walk from the manager (initiator) to the i-th queue, and E is again to be minimized.

Elsewhere my colleague Lundin and I have discussed the prevalence of organizations where the value of d varies from queue to queue.(13) As an illustration, consider the typical team teaching plan, where a team leader is both an intermediate and a classroom teacher.

To model this circumstance, we will suppose that Q is partitioned into the mutually exclusive and exhaustive subsets Q_i and Q_j by the criterion

$d_i \neq d_j$. The objective function becomes

$$E = \frac{n}{p} \sum_{i=1}^{p} (1 - k^{d_i})^{-1} e_i + \frac{n}{n-p+1} \sum_{j=p+1}^{n} (1 - k^{d_j})^{-1} e_j .$$

Collecting terms, we have

$$E = \sum_{h=1}^{n} e_h - \left[\frac{n}{p} \sum_{i=1}^{p} e_i \, k^{-d_i} + \frac{n}{n-p+1} \sum_{j=p+1}^{n} e_j \, k^{-d_j} \right].$$

The objective function is a minimum where the bracketed terms on the right-hand side of the equation are a minimum.

Let us now prove the following

Theorem (Kulilowski's "Second Theorem"(14)):

If $d_i > d_j$, then E is a minimum only if $e_i < e_j$.

Proof: For $d_i > d_j$, either $e_i \geq e_j$ or $e_i < e_j$. If the former, suppose $e_i = e_j$. Then

$$\sum_{h=1}^{n} e_h - E = \sum_{i=1}^{n} k^{-d_i} k^{-d_j} (k^{d_i} + k^{d_j}) e_i = \sum_{j=1}^{n} k^{-d_i} k^{-d_j} (k^{d_i} + k^{d_j}) e_j .$$

If $e_i < e_j$, then

$$\sum_{h=1}^{n} e_h - E = \sum_{i=1}^{p} \sum_{j=p+1}^{n} k^{-d_i} k^{-d_j} (k^{d_i} e_j + k^{d_j} e_i).$$

Since

$$\sum_{i=1}^{p} \sum_{j=p+1}^{n} k^{-d_i} k^{-d_j} (k^{d_i} e_j + k^{d_j} e_i) < \sum_{i=1}^{n} k^{-d_i} k^{-d_j} (k^{d_i} + k^{d_j}) e_i ,$$

the theorem is proved.

So far as the managerial problem in the case of varying values of d can be considered an assignment problem, the manager will optimize the organization's performance in terms of the objective function E if he requires those queues be relatively most efficient (least costly) which are most distant.

Confronting an organization where d varies, can the researcher expect the cost function of the organization to have a unimodal or a multimodal cost distribution? Clearly it will have a multimodal distribution, which reflects the underlying structural differentiation of queues. The local cost functions cannot be taken as a surrogate of the organizational cost function, and vice versa. In spite of this circumstance, we find in the vast majority of research on effectiveness in large-scale systems no attempt to block on structural and organizational factors. While this is clearly a specification error, in the case of the descriptive approach to benefit-cost analysis discussed above, it is an unavoidable error.

FOOTNOTES

(1) For a comprehensive discussion of the various modes of economic analysis in education, cf. S. Temkin, A Comprehensive Theory of Cost-Effectiveness, Philadelphia (1970).

(2) Cf. J. A. Thomas, "Cost-Benefit Analysis and the Evaluation of Educational Systems," Proceedings of the 1968 Invitational Conference on Testing Problems, Princeton (1969), p. 94.

(3) Cf. Thomas, op. cit. p. 96.

(4) H. J. Hartley, "PPBS and Cost-Effectiveness Analysis," Educational Administration Quarterly V (1969), p. 66.

(5) Cf. G. H. Fisher, "The Role of Cost-Utility Analysis in Program Budgeting," Program Budgeting, ed. D. Novick, Cambridge, Massachusetts (1965), p. 76.

(6) Cf. "Off the Editor's Chest," Consumer's Research Bulletin (February, 1950), pp. 18-20; also, M. Kaplan, "The Consumers Union Model," Evaluation: Processes and Practices, Selected Papers from the Conference for the Evaluation of Instructional Materials, Washington, D. C. (1968), pp. 45-46.

(7) Cf. F. Harary, R. Z. Norman, and D. Cartwright, Structural Models, New York (1965), pp. 283ff; also, A. J. Blikle, "Investigations on Organizations of Production Processes with Tree Structure," Mathematical Systems Theory and Economics, ed. H. W. Kuhn and G. P. Szegö, Vol. II, Berlin (1969), pp. 421ff on trees.

(8) We should note that we are making a heroic assumption that costs (and benefits) for all $x \notin Q$ are zero.

(9) D. Krathwohl, "A Paradigm for the Development of Research Designs." Presented at the American Educational Research Association, 1970 Annual Meeting, Minneapolis (1970).

(10) Cf. G. E. Lundin and G. A. Welty, "Relevance of a Managerial Decision-Model to Educational Administration." Presented at the American Educational Research Association, 1970 Annual Meeting, Minneapolis (1970). Available as ERIC Ed-041-356.

(11) Cf. O. Morgenstern, Prolegomena to A Theory of Organization, Santa Monica (1951), pp. 95ff on size and height relationships.

(12) Cf. A. Downs, Inside Bureaucracy, Boston (1967), p. 118.

Footnotes - continued

(13) G. Welty and E. Lundin, "Management Models and Large-Scale Structures." Presented at the Association for Educational Data Systems, 1970 Annual Convention, Miami Beach (1970). Available as ERIC ED-041-367. Also, G. E. Lundin and G. A. Welty, "Management Models and Educational Evaluation," Journal of Research and Development in Education III, (1970), pp. 44-45.

(14) The only proof of this theorem that I know is in R. Kulikowski, "Synthesis and Optimum Control of Organization in Large Scale Systems," Archiwum Automatydi i Telemechaniki 12 (1967), a relatively inaccessable East European journal. Hence, I have provided a proof of my own, which can be generalized from a two-level heirarchy to an n-level heirarchy by mathematical induction.